MATH 135 Lecture Notes Section 3.3 and 3.4 — Derivatives and Instantaneous Rates of Change
September 19, 2006

The definition of derivative is found in box 3-6 on page 82. The paragraph above the box reads:

> In the previous section, we saw that the slope of a line tangent to the curve y = *f(x)* at P(x,y) was found as the limiting value of the ratio $\Delta y/\Delta x$ as Δx approaches zero. Formally this limiting value of the ratio $\Delta y/\Delta x$ is known as the derivative of the function. Therefore the derivative of a function *f(x)* is defined as

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

There are several different ways to write the derivative. The four most common ones are given in the middle of page 84.

$$y' \qquad D_x y \qquad f'(x) \qquad \frac{dy}{dx}$$

We will usually use the last two. Read $\frac{dy}{dx}$ as "the derivative of *y* with respect to *x*."
The Greek letter "Δ" (delta) corresponds with the Roman letter "d" for difference. Thus the notation suggests that the derivative is the limit of the ratio of differences, Δy and Δx, and we can write

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

Most of the examples in Section 3.3 involve using the delta process to compute derivatives. We are going to study only Example 2 on page 83. This example shows that when $y = 6x - 2x^3$, $\frac{dy}{dx} = 6 - 6x^2$.

[Notice that in using the delta process we had to find the cube of a binomial:
$(x+\Delta x)^3 = x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3$]

In class we will explore the meaning of this by graphing the function Y1 = 6X – 2X^3 in the window Xmin = -4.7, Xmax = 4.7, Xscl = 1, Ymin = -10, Ymax = 10, Yscl = 1).

We will then display the graph and use the menu 2ndCALC to estimate dy/dx at the points where x takes on these values -2, -1.5, -1, .5, 0, .5, 1, 1.5, 2. Notice how positive values of dy/dx correspond with points where the function is increasing (i.e. the graph is rising). Similarly negative values of dy/dx correspond with points where the function is decreasing (i.e. the graph is falling). When dy/dx = 0 (as it is at two of the points), the graph is neither rising nor falling.

Another way the calculator can be used to find an estimate for the derivative is shown in Figure 3-21 on page 84. The calculator feature is called "nDeriv" for "numerical derivative." From the home screen find nDeriv by pressing MATH and scrolling down to 8 and pressing Enter. You then enter the function, the variable with respect to which you are finding the derivative (usually x), and the value at which you want to find the derivative, separated by commas. Thus as shown in Figure 3-21 nDeriv (6X-2X^3,X,-2) asks the calculator to find the derivative of the function $y = 6x - 2x^3$ with respect to the variable x for $x = -2$. The calculator returns an approximation to the actual answer which is -18. Now check this result two ways:

(1) Calculate the value of $\frac{dy}{dx} = 6 - 6x^2$ when -2 is substituted for x.

(2) Display the graph of Y1 = 6X – 2X^3 . TRACE to the point (-2, 4) and find the derivative by selecting 2ndCalc 6(dy/dx) and then pressing Enter.

Section 3.4 discusses the fact that the derivative represents an instantaneous rate of change. For example if s represents the displacement of an object moving along a straight line and t represents time, s is a function of t. The instantaneous velocity v at a particular time is defined in box 3-7 on page 87.

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

When t is given in seconds and s is given in feet, an object that is dropped from a position at rest near the surface of the earth travels a distance of $s = 16t^2$.

[Caution: the book uses s as a **variable** to represent distance or displacement and s as a **label** to represent seconds, a unit of time. Don't be confused!]

Example 3 (page 87) shows how to find the instantaneous velocity after when t = 4 seconds by a process of numerical approximation, similar to Example 1 on page 78. Example 4 (page 88) shows how to find the instantaneous velocity with the delta process.

Exercises: page 85: 1, 5, 9, 13 (use the delta process)

For exercise 9 above Graph Y1 = X^2 – 7X in the window Xmin = -9.4, Xmax = 9.4, XScl = 1, Ymin = -15, Ymax = 15, Yscl =5.
Use 2ndCalc to find dy/dx for X = 0, 1, 2, 3, 4, 5, 6, 7. Check your results by substituting into the expression for dy/dx found in Exercise 9.
For what value of x is dy/dx = 0?

Page 86: 25, 27 (use only the calculator: nDeriv and 2ndCalc dy/dx)
Page 89: 5, 7, 9, 11